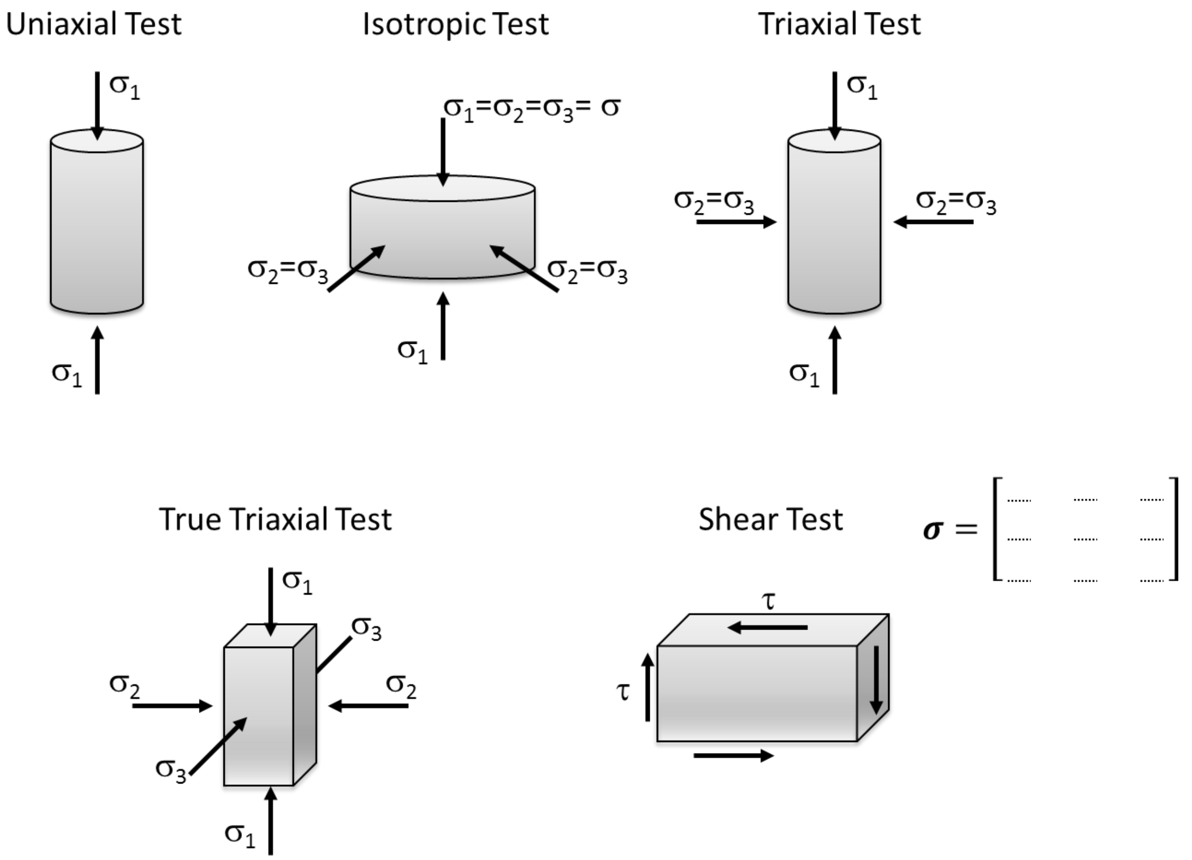
Exercise 2 – 02.10.2025

Elasticity

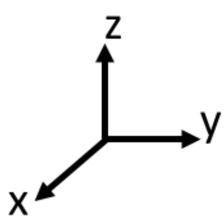
1. Stress tensors definitions

Part 1: Qualitative cases

Define the stress tensor for the following cases representative of the most common laboratory test conditions:

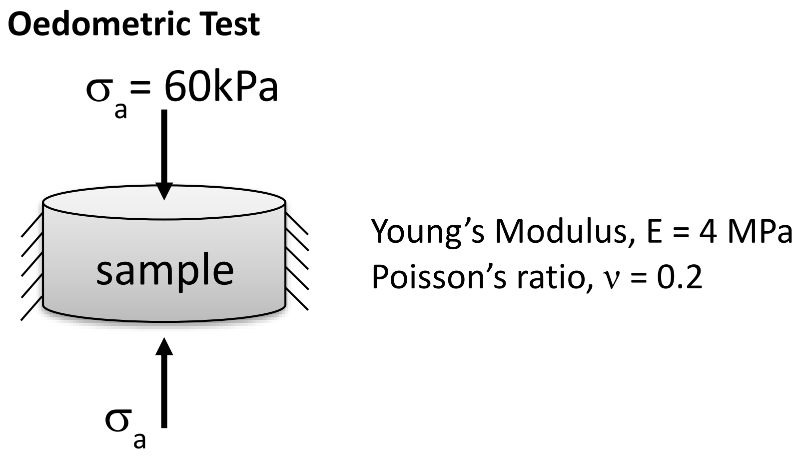


Consider the following coordinate system:



Part 2: Computation example

Consider an oedometric test where the sample is subjected to a vertical axial stress of 60 kPa. Given the properties of the tested material, calculate the different stress components of the stress tensor. (Use the elastic relationship between the stress and strain components)



2. Parameter determination for an undrained elastic material

A conventional undrained triaxial compression test, with the cell pressure σ_c, held constant, is carried out on a sample of stiff overconsolidated clay. The stress-strain relationship is found to be linear up to failure, so it is deduced that the clay behaves as an isotropic elastic material. The Biot coefficient can be considered to be 1 (due to incompressible grains). Consider a back pressure (initial pore water pressure) equal to 0.

Tip: *to compute G, consider the following equation* $\Delta q = 2G(\Delta\varepsilon_{11} - \Delta\varepsilon_{33})$ *obtained through the relationship between deviatoric stress and invariants (derivation is shown in annex 1).*

Part 1

After an axial strain $\Delta\varepsilon_a$ = 0.8%, the corresponding deviatoric stress is q = Δq = 80*kPa*.

Calculate the corresponding values of:

- Mean total stress Δp,
- Mean effective stress $\Delta p'$
- Pore water pressure Δp_w
- Radial effective stress $\Delta \sigma'_r$
- Axial total stress $\Delta \sigma_a$
- Axial effective stress $\Delta \sigma'_a$
- Radial deformation $\Delta \varepsilon_r$
- Undrained Young's modulus E_u
- Undrained shear modulus G

Part 2

At this time, the axial stress and cell pressure are kept constant, and the sample is allowed to drain so that the pore pressures dissipate and the sample undergoes a volumetric strain $\Delta\varepsilon_v$ = 0.25%

Calculate the values of $\Delta p_w, \Delta\sigma'_a, \Delta\sigma'_r, \Delta q, \Delta p', \Delta\varepsilon_a, \ \Delta\varepsilon_r$

Calculate the value of bulk modulus K.

Appendices

Annex 1

Undrained shear modulus G from deviatoric stress and invariants:

- The deviatoric stress q is related to the invariant I_{2D} by the shear modulus G, according to the following equation:

$$q = 2\sqrt{3}G\sqrt{I_{2D}}$$

- This equation is obtained from the relationship between deviatoric stress q and the second invariant J_{2D} of the deviatoric stress tensor s_{ij}:

$$J_{2D} = \frac{1}{6}[(\sigma_1 - \sigma_3)^2 + (\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2] = \frac{1}{6}[2(\sigma_1 - \sigma_3)^2] = \frac{1}{3}[(\sigma_a - \sigma_r)^2]$$

$$\mathrm{q} = \sqrt{3\mathrm{J}_{2\mathrm{D}}}$$

- Considering the elastic constitutive relationship between deviatoric stress tensor s_{ij} and deviatoric strain tensor e_{ij}:

$$s_{ij} = 2Ge_{ij}$$

- J_{2D} can be written as:

$$\sqrt{\mathrm{J}_{2\mathrm{D}}} = 2\mathrm{G}\sqrt{\mathrm{I}_{2\mathrm{D}}}$$

where, in triaxial conditions, I_{2D} is second invariant of the deviatoric strain tensor e_{ij} :

$$\mathrm{I}_{2\mathrm{D}} = \frac{1}{6}[(\varepsilon_1 - \varepsilon_3)^2 + (\varepsilon_1 - \varepsilon_2)^2 + (\varepsilon_2 - \varepsilon_3)^2] = \cdots = \frac{1}{3}(\varepsilon_a - \varepsilon_\mathrm{r})^2$$

- Finally, we can write $q = 2G(\varepsilon_{11} - \varepsilon_{33})$, and $\Delta q = 2G(\Delta\varepsilon_{11} - \Delta\varepsilon_{33})$